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HUDSON PARK HIGH SCHOOL

GRADE 12

SUBJECT: MATHEMATICS PAPER 1

DATE: JUNE 2015 **TOTAL: 150 MARKS**

EXAMINER: C Selkirk **TIME: 3 HOURS**

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately.
3. A Diagram Sheet is provided. Please detach it and use it.
4. NB: Please STAPLE your submission in the following order:

   Foolscap answer pages (on top)

   Diagram sheets (middle)

   Question paper (bottom)

5. Employ relevant formulae and show all working out.
6. (Non programmable and non graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. Start each new Question at the top of a new side of paper.

25

Question 1 (~~28~~ marks)

1.1 Solve for x:

1.1.1 $-x^2 - 5x = -6$ (3)

1.1.2 $-3x^2 + 4x + 5 = 0$ (4)

1.1.3 $\frac{2x^2}{x+2} \leq 2$ (5)

1.1.4 $2^x - \frac{12}{2^x} = 4$ (5)

1.2 Solve for x and y

$2y - x = 3$

$x^2 - 3xy - y^2 = 27$ ~~(7)~~ 4

1.3 Simplify **without using a calculator:**

$$\frac{3^{3001}.3^2}{27^{1001} - 3^{3002}}$$ (4)

~~[28]~~ 25

Question 2 [36 marks]

2.1 Write the following series in sigma notation:

30 + 28 + 26 + ... − 18 (3)

2.2 The sum of the first n terms of a sequence is given by $S_n = 3^{n+1} - 6$

2.2.1 Determine the sum of the first 12 terms. (1)

2.2.2 Determine the 12th term. (2)

2.3 The sum of the first five terms of an arithmetic series is zero. The fifth term is 4. Determine the first term and the common difference. (5)

2.4 -15; - 29; - 43; .... are the first three first differences of a quadratic sequence, whose 30th term is − 6102. Determine the nth term of the quadratic sequence. (6)

2.5 Given the following series:

$$17 + 3 + 15 - \frac{3}{2} + 13 + \frac{3}{4} + \ldots\ldots\ldots\ldots \frac{-3}{512}$$

2.5.1 How many terms are in this series? (4)

2.5.2 Evaluate the series. (5)

2.6 A man stands on a wall 6 m high and drops a bouncing ball. Each bounce is 9/10 as high as the previous bounce. What is the distance the ball bounces before coming to rest? (3)

2.7 Using an infinite geometric series, convert the recurring decimal $1,\dot{3}\dot{6}$ to an improper fraction. (3)

2.8 Prove that, the sum of the geometric series below, can be given by the formula $S_n = \frac{a(r^n-1)}{r-1}$

$T_1 + T_2 + T_3 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ (4)

[36]

Question 3[16 marks]

3.1 A motor car costing R 240 000 depreciates at a rate of 8% per annum on a reducing balance basis. Calculate how long it would take for the car to depreciate to a value of R 95 000. (3)

3.2 A house costs R 1,3 million. 20% is paid in cash and the balance is paid using a bank loan.

3.2.1 Calculate the monthly repayments if the loan is repaid over a period of 20 years by equal monthly payments and the interest rate is 12% p.a. compounded monthly. (4)

3.2.2 Calculate the total amount of money paid for the house over the 20 years. (1)

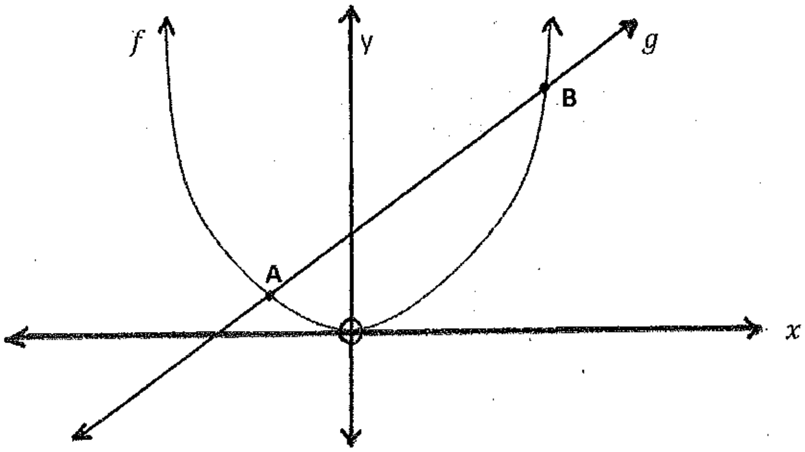
3.2.3 Calculate the balance remaining on the loan at the end of 10 years. (3)

3.3 Rebecca deposits R 7 000 into an account paying 14 % per annum compounded half-yearly. Six months later she deposits R 400 into the account. Six months after this, she deposits a further R 400 into the account. She then continues to make half yearly deposits of R 400 into the account for a period of nine years from the initial deposit of R 7 000. Calculate the value of her savings immediately after her final deposit of R 400 at the end of nine years. (5)

[16]

Question 4 (22 marks)

The graphs of $f(x) = 2x^2$ and $g(x) = x + 3$ are sketched below. A and B are the points of intersection of f and g.



4.1 Determine the coordinates of A and B. (6)

4.2 Give the coordinates of two points of intersection of f^{-1} and g^{-1}. (2)

4.3 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.4 Determine the equation of f^{-1} in the form $y = \ldots$ (2)

4.5 Use diagram sheet A and draw the graphs of f^{-1} and g^{-1} on the same set of axes. (2)

4.6 Use your graphs to answer the following questions. For which values of x:

4.6.1 are f and g both increasing? (2)

4.6.2 is $f(x).g(x) \leq 0$? (2)

4.6.3 must the domain of f be restricted so that f^{-1} is a function? (2)

4.7 Find the average gradient of $f(x)$ between the origin and point A. (2)

[22]

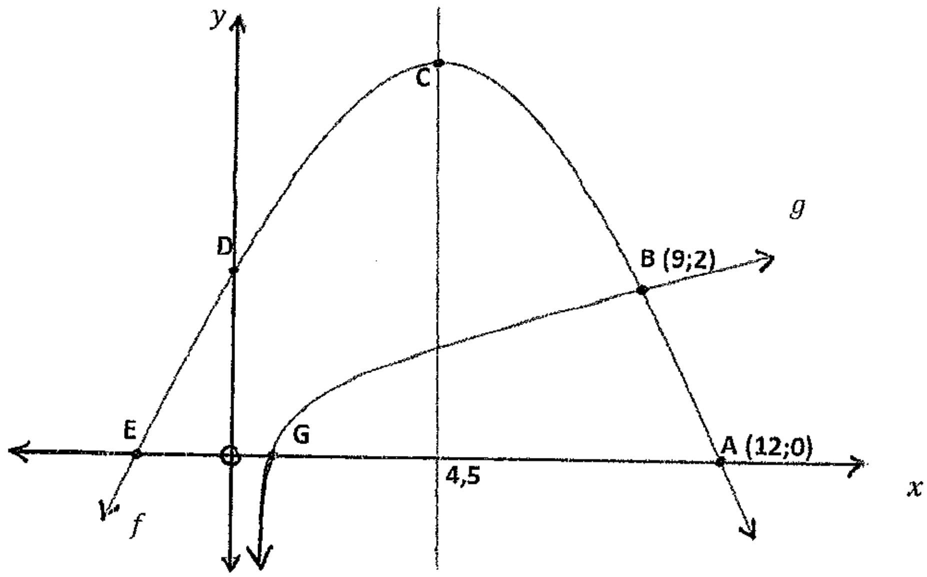
18

Question 5 (~~16~~ marks)

In the sketch, the graphs of the functions given by $f(x) = ax^2 + bx + c$ and $g(x) = \log_m x$ are represented.

A (12;0) is an x-intercept of f and $x = 4{,}5$ is the axis of symmetry of f.

B (9;2) is the point of intersection of f and g



5.1 Determine the value of m (2)

5.2 What are the coordinates of G? (2)

5.3 Write down the domain of g ~~(1)~~ 2

5.4 Determine the equation of g^{-1} in the form $y =$ (2)

5.5 Write down the equation of h if h is obtained by shifting g^{-1} 2 units to the left. (1)

5.6.1 Explain why the coordinates of E will be (-3;0). (1)

5.6.2 Now, determine the equation of the parabola f and hence show that

$a = \frac{-1}{18}$; $b = \frac{1}{2}$ and $c = 2$. (3)

5.7 . Write f in the form $f(x) = a(x-p)^2 + q$, using the values from 5.6.2. (3)

5.8 Use your answer to QUESTION 5.7 and the graph to explain why the equation $f(x) - 4 = 0$ will have no real roots.

~~(1)~~ 2

~~[16]~~ 18

Question 6 (11 marks)

6.1 Draw a neat sketch graph of $f(x) = \frac{-3}{x+2} + 3$, showing all intercepts with axes. (5)

6.2 For the graph above, write down the equations for:

6.2.1 the axes of symmetry. (2)

6.2.2 the asymptotes. (2)

6.3 What is the domain of f? (2)

[11]

Question 7 (8 marks)

7.1 $(x-1)$ is a factor of $h(x) = x^3 + (q-4)x^2 \cdot (3-4q)x + 3$. Determine the value of q. (3)

7.2 Consider the expression $2x^3 + x^2 - 5x + 2$

7.2.1 Show that $(2x-1)$ is a factor of the above expression. (2)

7.2.2 Hence factorise $2x^3 + x^2 - 5x + 2$ completely. (3)

[8]

Question 8 (9 marks)

8.1 Find $f^{I}(x)$ from first principles if $f(x) = \frac{3}{x}$ (6)

8.2 Hence, find $f^{I}(3)$ and explain what this answer means. (3)

[9]

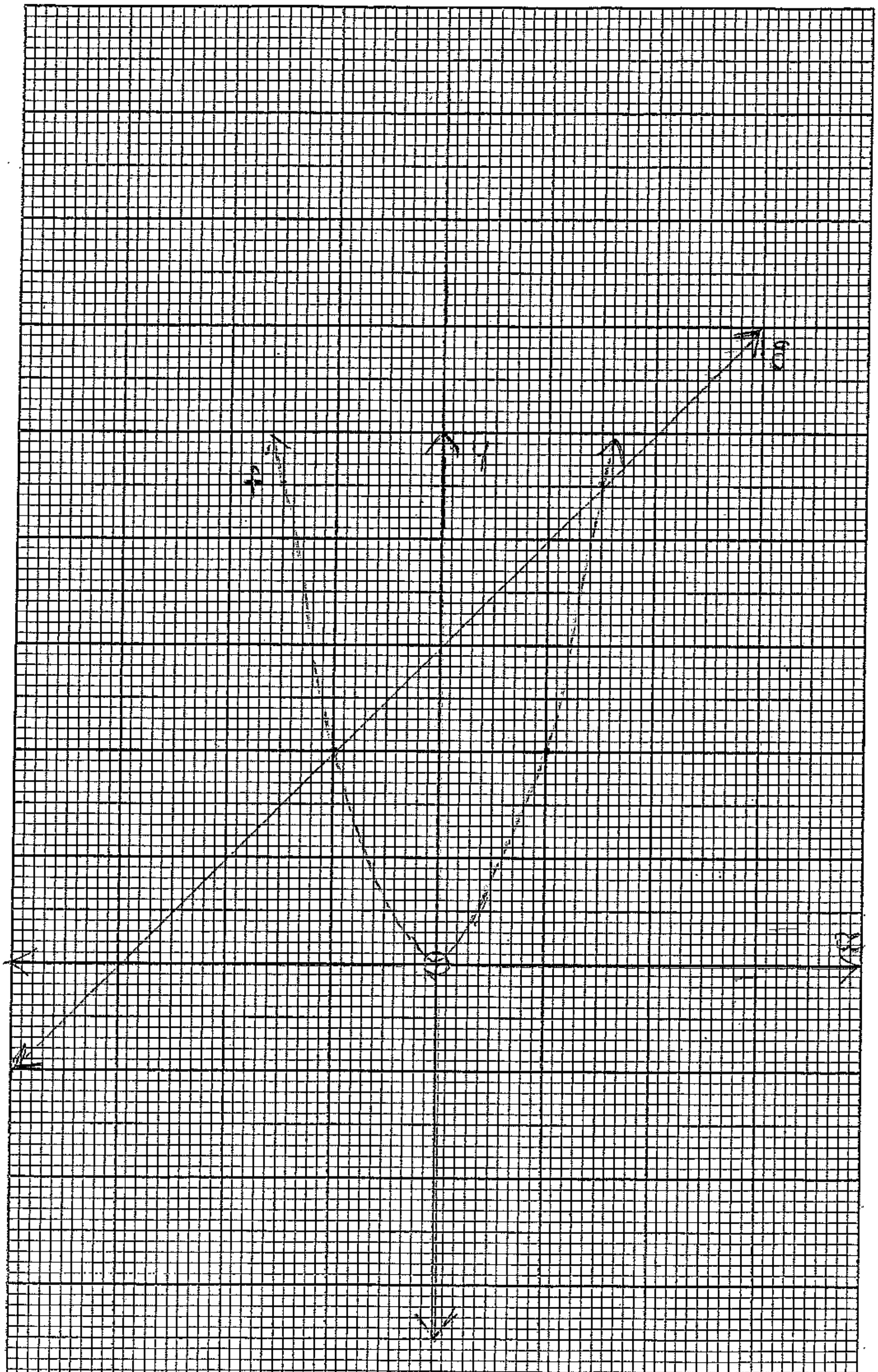
Question 9 (~~4~~ 5 marks)

If 3 cards are drawn from a pack of 52 cards without replacing them, determine the probability that they are all face cards (King, Queen or Jack). Remember each one of the four suits in a pack of cards has 3 face cards. Do a tree diagram to help you calculate the probability. ~~[4]~~ 5

TOTAL: 150 MARKS

Diagram Sheet A

4.5



5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

